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The Casimir effect in rugby-ball type flux compactifications

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Abstract

We discuss volume stabilization in a 6D braneworld model based on 6D supergravity theory. The internal space is compactified by magnetic flux and contains codimension two 3-branes (conical singularities) as its boundaries. In general the external 4D spacetime is warped and in the unwrapped limit the shape of the internal space looks like a 'rugby ball'. The size of the internal space is not fixed due to the scale invariance of the supergravity theory. We discuss the possibility of volume stabilization by the Casimir effect for a massless, minimally coupled bulk scalar field. The main obstacle in studying this case is that the brane (conical) part of the relevant heat kernel coefficient (a_6) has not been formulated. Thus as a first step, we consider the 4D analog model with boundary codimension two 1-branes. The spacetime structure of the 4D model is very similar to that of the original 6D model, where now the relevant heat kernel coefficient is well known. We derive the one-loop effective potential induced by a scalar field in the bulk by employing zeta function regularization with heat kernel analysis. As a result, the volume is stabilized for most possible choices of the parameters. Especially, for a larger degree of warping, our results imply that a large hierarchy between the mass scales and a tiny amount of effective cosmological constant can be realized on the brane. In the non-warped limit the ratio tends to converge to the same value, independently of the bulk gauge coupling constant. Finally, we will analyze volume stabilization in the original model 6D by employing the same mode-sum technique.

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(Some figures in this article are in colour only in the electronic version)

1. Introduction

There are longstanding problems in phenomenology and cosmology. One of them is why gravity is so weak in comparison with the electroweak interactions (in other words why Planck scale $M_{\rm Pl} \sim 10^{19}$ GeV is much larger than that of electroweak interaction $M_{\rm EW} \sim 10^3$ GeV), i.e. the *hierarchy problem*. Another problem is why the energy density of dark energy which dominates the present universe (assuming that its origin is vacuum energy of quantum fields) $\rho_{\rm Vac} \sim (10^{-3}\ {\rm eV})^4$ is much smaller than that of gravitational scale $M_{\rm Pl}^4$, which is naturally expected from the standard model and is known as the *cosmological constant problem*.

In this paper, we focus on the Casimir effect in a 6D braneworld model whose internal space is compactified by a magnetic flux [1]. 6D flux compactification models have attracted particular attentions in recent years since they may help to resolve the above problems. The model corresponds to a simple realization of the scenario of the 'large extra dimensions' (in 6D) [2], where one can choose the gravitational energy scale M_6 as large as the electroweak scale $M_6 \sim M_{\rm EW}$. What is essential is the presence of the brane, where standard model particles can be localized. At the same time, the extra dimensions could be detected in the future gravity test, since their size may be 0.1 mm scales [2].

Also another important point is that the gravitational property of the codimension two brane, i.e. 3-brane in 6D spacetime, is suitable for resolution of the cosmological constant problem, since the vacuum energy of the brane does not curve our 4D spacetime and just change the deficit angle in the bulk [3][1].

Keeping the above properties in mind, the Casimir effect in 6D brane spacetime may play an important role. An important point is that $M_{\rm Pl} \sim 10^{16} M_{\rm EW}$ and $\rho_{\rm Vac}^{1/4} \sim 10^{-16} M_{\rm EW}$. Thus in these two hierarchies there is similarity and these problems may be solved at the same time, if the theory could give the common factor as large as 10^{16}. The Casimir effect in two compactified dimensions may be able to give the common factor. Imagine a spacetime with two extra dimensions whose size is assumed to be stabilized at a characteristic scale a. Then, the effective 4D Planck mass is given by $M_{\rm Pl}^2 = a^2 M_6^4$ [2], where we assume $M_6 \sim M_{\rm EW}$. Casimir energy density induced on the brane by fields living in the internal space is roughly given by $\rho_{\rm Cas} \sim a^{-4}$. Thus, we get $\rho_{\rm Cas}/M_6^4 \sim (M_6/M_{\rm Pl})^4$. If the Casimir energy density $\rho_{\rm Cas}$ plays the role of the current dark energy density $\rho_{\rm Vac}$, we can get the desired ratio (see also [5]). The problem is whether the ratio is really obtained from the set-up of the 6D braneworld. We focus on the Casimir effect in a concrete model of 6D braneworld with a warped compactification based on the 6D Nishino–Sezgin (Salam–Sezgin) supergravity [6].

In this model, the 2D internal space is supported by a magnetic flux and contains codimension two branes (conical singularities) as boundaries. In the special limit, the shape of the internal space looks like a 'rugby ball'. The size of the internal space is not fixed due to the scale invariance of the underlying theory and we discuss the volume stabilization by the Casimir effect[2]. We take the approach to perform the zeta function regularization combining with the heat kernel analysis. The main obstacle is that the brane (conical) contribution to the relevant heat kernel coefficient is not formulated. Thus, instead of the original 6D model, as the fist step we consider the 4D version of the warped compactification. Then, we shall discuss the possibility of volume stabilization in the original 6D model based on the mode-sum technique developed in the study of 4D model (see [8–10]).

[1] It has also been suggested that the so-called 'self-tuning' mechanism of the cosmological constant does not work due to hidden fine-tunings [4].

[2] For another way of the volume stabilization in such a model, see [7].

2. 6D warped compactifications

2.1. Solution

We consider a 6D Einstein–Maxwell-dilaton theory with a non-vanishing scalar potential [1, 6] as

$$S_6 = M_6^4 \int \mathrm{d}^6x\sqrt{-g}\left(\frac{1}{2}R - \frac{1}{2}\partial_A\varphi\partial^A\varphi - \frac{1}{4}\,\mathrm{e}^{-\varphi}F_{AB}F^{AB} - 2g_6^2\,\mathrm{e}^{\varphi}\right), \quad (1)$$

where φ is a dilaton field, F_{AB} represents a $U(1)$ gauge field strength and g_6 is the $U(1)$ gauge coupling constant. This theory corresponds to the bosonic part of the Nishino–Sezgin (Salam–Sezgin) 6D supergravity [6], whose other fields contents can be chosen to be vanishing consistently. Hereafter we basically set $M_6^4 = 1$ for simplicity.

This theory contains a series of solutions of warped compactifications [1] ;

$$\mathrm{d}s^2 = h(\rho)\,\mathrm{d}\theta^2 + \frac{\mathrm{d}\rho^2}{h(\rho)} + (2\rho)\eta_{\mu\nu}\,\mathrm{d}x^\mu\,\mathrm{d}x^\nu, \qquad h(\rho) = \frac{g_6^2}{2\rho^3}\left(\rho_+^2-\rho^2\right)\left(\rho^2-\rho_-^2\right),$$
$$\varphi(\rho) = -\ln(2\rho), \qquad F_{\theta\rho} = -\frac{g_6\rho_+\rho_-}{\rho^3}, \quad (2)$$

where two 3-branes are located at $\rho = \rho_\pm$. For later convenience, we define a new parameter $\alpha = \rho_-/\rho_+$ which controls shape (warping) of the internal space.

The braneworld action is given by $S_\pm = -\int \mathrm{d}^4x\sqrt{-h}\sigma_\pm$, respectively, where $\sigma_\pm$ denotes the brane tensions, which are related to the conical deficit angles by $\sigma_\pm = M_6^4\delta_\pm$. h_{ab} is the brane-induced metric. The deficit angles are related to α as $(2\pi-\delta_+)/(2\pi-\delta_-) = \alpha^2$. Once the brane tensions, $\sigma_\pm$ are specified, then the bulk shape α is also fixed. We now regard α and $\omega := 2\pi - \delta_+$ as free parameters, instead of $\sigma_\pm$, along with g_6. So we shall use (+)-brane as a reference brane. The remaining modulus is the absolute size of the bulk, ρ_+. To discuss the modulus dynamics, we take the moduli approximation, namely assuming that $\rho_+ \to \rho_+(x^\mu)$. After integrating over the extra dimensions and and redefining the modulus as $\chi_6(x^\mu) = \sqrt{\left(8\pi\omega M_6^4/g_6^2\right)\rho_+}$ we obtain the canonical form of the modulus kinetic term.

2.2. One-loop effective potential

Next, we introduce a massless, minimally coupled scalar field and work in the Euclideanized space. The action for the massless scalar field perturbations is given by

$$S_{\mathrm{scalar}} = \frac{1}{2}\int \mathrm{d}^6x\sqrt{-g}\phi\Delta_6\phi. \quad (3)$$

The one-loop effective action for a massless minimally coupled scalar field is given by $W_6 = (1/2)\ln\det(-\Delta_6)$, where Δ_6 is 6D Laplacian. W_6 needs to be regularized and renormalized. For this purpose, we introduce the (integrated) zeta function, which is given by the mode summatio

$$\zeta(s,\Delta_6) = \int \mathrm{d}^4x\sum_{m,n}\int\frac{\mathrm{d}^4k}{(2\pi)^4}\frac{1}{\lambda^{2s}}, \quad (4)$$

where the eigenvalues are defined by $\Delta_6\phi_\lambda = -\lambda^2\phi_\lambda$. $m = 0, 1, 2, \ldots, n = 0, \pm1, \pm2, \ldots$ represents the angular quantum number and k corresponds to the usual 4-momentum. (We now are working in the Euclideanized space.) The renormalized scalar field effective action can be written in terms of the analytically continued zeta function $W_{6,\mathrm{ren}} = -(1/2)\zeta'(0,\Delta_6) - (1/2)\zeta(0,\Delta_6)\ln\mu^2$. By integrating over the internal dimensions, the 4D effective potential is $W_{6,\mathrm{ren}} = \int\left(\mathrm{d}^4x\rho_+^2\right)V_{6,\mathrm{eff}} = \int \mathrm{d}^4\tilde{x}\,V_{6,\mathrm{eff}}$. For brevity, from now on we shall omit the subscript 'ren'. One strategy to evaluate the one-loop effective action and the effective potential is to define a continuous conformal transformation (parameterized by ϵ)

$$d\tilde{s}^2_{6,\epsilon} = e^{2(\epsilon-1)\Omega}\, ds^2_6, \qquad \Omega = \tfrac{1}{2}\ln(2\rho), \tag{5}$$

where for $\epsilon = 1$ we have the original metric, which we shall denote as $\Delta_{6,\epsilon} = \Delta_6$. The classical action of this scalar field is changed under a conformal transformation equation (5): $S_{\text{scalar}} = -(1/2)\int d^6x\sqrt{g}\phi\Delta_6\phi = -(1/2)\int d^6x\sqrt{\tilde{g}}\tilde{\phi}(\tilde{\Delta}_6 + E_6(\epsilon))\tilde{\phi}$, where $E_6(\epsilon) = -4(\epsilon-1)^2\tilde{g}^{ab}\nabla_a\Omega\nabla_b\Omega + 2(\epsilon-1)\tilde{\Delta}_6\ln\Omega$. Then, the effective potential can be written as

$$V_{6,\text{eff}}(\alpha,\omega,g_6,\mu;\rho_+) = \frac{A_6(\alpha,\omega,g_6) - B_6(\alpha,\omega,g_6)\ln(\mu^2\rho_+)}{\rho_+^2}, \tag{6}$$

where we define

$$\begin{aligned}\int d^4x\, A_6(\alpha,\omega,g_6) &= \int d^4\tilde{x}\,\frac{A_6(\alpha,\omega,g_6)}{\rho_+^2}\\ &= -\int_0^1 d\epsilon\, a_6\left(f = \frac{1}{2}\ln\left(\frac{2\rho}{\rho_+}\right)\right) - \frac{1}{2}\zeta'(0,\Delta_{6,\epsilon=0}),\\ \int d^4x\, B_6(\alpha,\omega,g_6) &= \int d^4\tilde{x}\,\frac{B_6(\alpha,\omega,g_6)}{\rho_+^2} = \frac{1}{2}\zeta(0,\Delta_{6,\epsilon=0}).\end{aligned} \tag{7}$$

The term a_6 is given by the volume integration of linear combinations of cubic order curvature invariants (see [11]). Clearly, if $B_6(\alpha,\omega,g_6) > 0$, then the modulus effective potential has a minimum at $\rho_+^* = \mu^{-2}\, e^{(2A_6+B_6)/(2B_6)}$. It is straightforward to show that $\zeta(0,\Delta_6) = a_6(f=1)$. The problem, however, is that the concrete form of the conical heat kernel is not formulated. We mainly analyze the 4D analog model.

2.3. Phenomenological implications

2.3.1. Hierarchy problem. A way to resolve the hierarchy problem in braneworld set-up was first proposed in the 'large extra dimensions' scenario in [2]. In this scenario, the fundamental gravitational scale is not M_{Pl} but the higher-dimensional one (M_6 in 6D braneworld) and $M_6 \sim M_{\text{EW}}$. Then, the observed Planck scale is effectively given by $M_{\text{pl}}^2 \simeq (\rho_+(2\pi\omega)/g^2)M_6^4$. To get the observed value of reduced Planck scale, the size of extra dimension should be $(\rho_+)^{1/2} \sim 0.1$ mm. Thus now we ask whether volume stabilization at this scale can be realized in the present model. If we assume a brane localized field[3] whose mass is given by m^2 on either brane at $\rho_\pm$ then the observed mass scales are $m_+^2 = m^2$ and $m_-^2 = \alpha^2 m^2$. We now assume that $m_\pm^2 \sim M_{\text{EW}}^2$. Thus, the mass ratio between the field and the effective Planck mass is roughly given by $\left(m_\pm^2/M_{\text{pl}}^2\right) \simeq \left(\mu^2 m^2/M_6^4\right)\left(g_6^2/(2\pi\omega)\right) e^{-(2A_6+B_6)/(2B_6)}$. Assuming that the factor of $\left(\mu m/M_6^2\right)^2$ takes the optimal value of $\mathcal{O}(1)$ for the unification of all the fundamental energy scales in 6D, the effective mass ratio is characterized by

$$R(\alpha,\omega,g_6) := \frac{g_6^2}{2\pi\omega}\, e^{-(2A_6+B_6)/(2B_6)}|_{\rho_+=\rho_{+,*}}, \tag{8}$$

where we have used the value of $\rho_{+,*}$ at the stabilization. As is explained above, once the size of the internal space $\rho_+^{1/2}$ is stabilized at 0.1mm, then R has a value as $\sim 10^{-32}$.

[3] We implicitly assume that the brane has a extremely small, but finite thickness so that ordinary particles can be localized.

2.3.2. Cosmological constant problem. A characteristic property of 6D braneworld is that the tree level vacuum energy of the brane, i.e. brane tension, only changes the bulk deficit angle [3]. Thus, if the brane geometry is 4D Minkowski, then sudden changes of brane vacuum energy would not affect the brane geometry. Such a mechanism is called *self-tuning* mechanism. Note that there are several criticisms for such a self-tuning mechanism mainly because there could be hidden fine-tunings, e.g. due to the magnetic flux quantization condition in the bulk [4]. In the following discussions, we implicitly assume that such a mechanism works.

After volume stabilization, the effective potential (Casimir energy density) takes the value $V^*_{6,\mathrm{eff}}(\alpha,\omega,g_6) = -(1/2)\mu^4 B_6(\alpha,\omega,g_6)\,\mathrm{e}^{-(2A_6+B_6)/B_6}\big|_{\rho_+=\rho_{+,*}} (=\rho_{\mathrm{Cas}})$ and hence, the realized brane vacuum energy is almost completely determined by the renormalization. The renormalization scale μ could be chosen to be $\mu \sim M_6(\sim M_{\mathrm{EW}})$. Casimir energy density becomes negative. It could be uplifted, e.g. by taking fermionic contributions into account. We now, simply focus on the absolute value of the energy density.

3. Volume stabilization in the 4D warped compactifications

3.1. 4D warped compactification model

As mentioned in the previous section, there is no mathematical formulation of the conical contribution of the heat kernel coefficient a_6 as long as the authors are aware of. Thus, we focus on the 4D version of warped compactificaitions, where the relevant heat kernel a_4 is formulated. The action is given by

$$S_4 = M_4^2 \int \mathrm{d}^4x\sqrt{-g}\left(R - \frac{1}{2}\partial_A\varphi\partial^A\varphi - \frac{1}{8}\,\mathrm{e}^{-\varphi}F_{AB}F^{AB} - 4g_4^2\,\mathrm{e}^{\varphi}\right), \tag{9}$$

which has a series of warped flux compactification solutions

$$\begin{aligned} &\mathrm{d}s^2 = h(\rho)\,\mathrm{d}\theta^2 + \frac{\mathrm{d}\rho^2}{h(\rho)} + (2\rho)\left(-\mathrm{d}\tau^2 + \mathrm{d}x_2^2\right), \qquad h(\rho) = \frac{2g_4^2}{\rho}(\rho_+ - \rho)(\rho - \rho_-),\\ &\varphi(\rho) = -\ln(2\rho), \qquad F_{\theta\rho} = -\frac{g_4\rho_+\rho_-}{\rho^2}. \end{aligned} \tag{10}$$

We also set $M_4 = 1$. Then, the branes are corresponding to the strings, lying at $\rho = \rho_\pm$.

We also consider Casimir effect by a massless, minimaly coupled scalar field $S_{\mathrm{scalar}} = -(1/2)\int \mathrm{d}^4x\sqrt{-g}\phi\Delta_4\phi$. The discussion in deriving the explicit form of the effective potential is very similar to the case of the 6D model. The one-loop effective potential is given by

$$V_{4,\mathrm{eff}}(\alpha,\omega,g,\mu;\rho_+) = \frac{A_4(\alpha,\omega,g_4) - B_4(\alpha,\omega,g_4)\ln(\mu^2\rho_+)}{\rho_+}. \tag{11}$$

The coefficients in equation (11) can be written in the form just as equation (7), by replacing a_6, $\mathrm{d}^4\tilde{x}$ and g_6 with a_4, $\mathrm{d}^2\tilde{x}$ and g_4, respectively. The zeta function $\zeta(s)$ is also defined as equation (4), by replacing Δ_6 and d^4k with Δ_4 and d^2k, respectively. However, we use the given mass spectrum in the unwrapped frame

$$(\tilde{\Delta}_4 + E(0))\tilde{\phi}_\lambda = -\lambda^2\tilde{\phi}_\lambda. \tag{12}$$

Here we assume that the mode functions are regular on both conical boundaries and we obtain the following mass spectrum:

$$\lambda^2 = k^2 + g_4^2\left[4m(m+1) + \frac{2|n|}{\omega}(2m+1)(1+\alpha) + \frac{4n^2\alpha}{\omega^2}\right]. \tag{13}$$

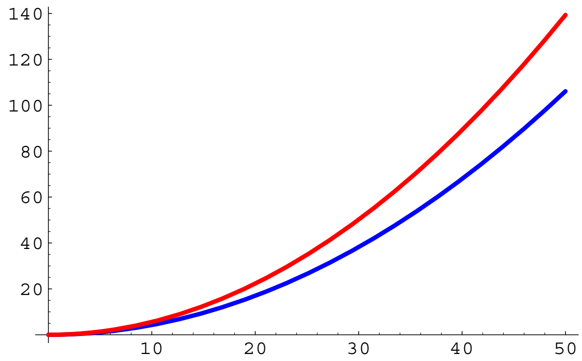

Figure 1. Numerical plot of the integrand of $\zeta(0)$ and $2B_4$ as functions of g_4 are shown for $\alpha = 1$ and $\delta_+ = 0.01$ in the 4D model. The red and blue curves correspond to the cases of $\zeta(0)$ and $2B_4$, respectively. We set $j_{\max} = n_{\max} = 50$.

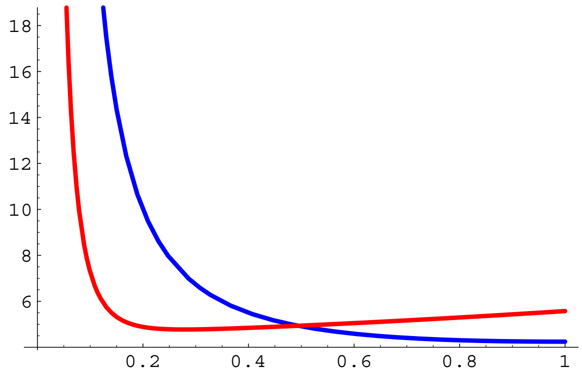

Figure 2. Numerical plot of the integrand of $\zeta(0)$ and $2B_4$ as functions of α are shown for $g_4 = 10$ and $\delta_+ = 0.01$ in the 4D model. The red and blue curves correspond to the cases of $\zeta(0)$ and $2B_4$, respectively. We set $j_{\max} = n_{\max} = 50$.

We evaluated the cocycle function by the integration of the heat kernel coefficient a_4, whose conical parts are known, and the derivative of the zeta function $\zeta'(0)$ via the summation of all the KK modes equation (13). For detailed forms of these coefficients, see [8–10]. We now derive the mass spectrum in the unwrapped frame. The solution for the radial mode of equation (12) can be written in terms of the hypergeometric functions.

As a consistency check of our results, in figures 1 and 2 we have plotted the integrands of $\zeta(0)$ and $B_4(\alpha, \delta_+, g_4)$, which are related to $a_4(f = 1)$, as functions of g_4 and α, respectively, for a fixed value of the deficit angle, δ_+. Note that both quantities are practically insensitive to δ_+. In the unwrapped frame, due to the properties of the heat kernel coefficients (see, e.g. [11]), the equation $\zeta(0) = a_4(f = 1)$ should be satisfied. Although they do not agree exactly, they do exhibit a similar behavior, namely increasing for larger g_4 and for smaller α. In [8] we confirmed that B_4 (and thus $a_4(f = 1)$) is independent of the value of ϵ [11], which characterizes the conformal transformation (as given in equation (5) for 6D model) and therefore, we believe that B_4 is correct. We have also carefully checked the convergence of the mode summations. Thus, it is the view of the author that a possible origin for the disagreement

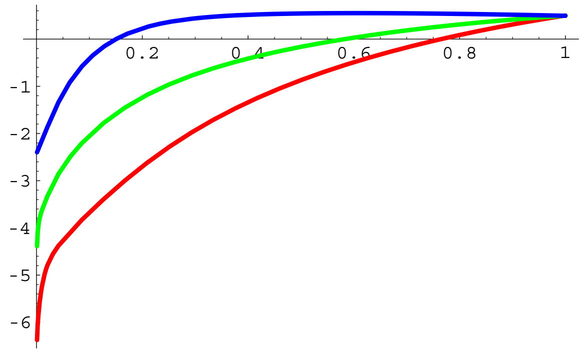

Figure 3. Numerical plots of $\log_{10} R(\alpha, \delta_+ = 0.01, g_4)$ as a function of α are shown for $g_4 = 0.5, 5, 50$ (the red, green and blue curves, respectively) in the 4D model. We set $j_{\max} = n_{\max} = 50$ as a conservative choice.

could be in our determination of the exact mode spectrum. By imposing vanishing conditions for the mode functions and their derivatives at the poles (branes), we were able to derive the mass spectrum in the rugby-ball frame. Our method to determine the mass spectrum is essentially based on the same arguments given in [12], where two conditions, normalizability and regularity at the poles were imposed. However, if only normalizability were imposed, this would allow for logarithmic divergences at the poles and this may well lead to additional modes in the eigenvalue spectrum, which could then reflect in the differences between figures 1 and 2. Regardless of this, the existence of such modes does not seem likely to significantly affect the qualitative behavior of our results.

3.2. Volume stabilization and implications

In the most of cases, we have shown that $B_4(\alpha, \omega, g_4)$ is positive and the volume is stabilized by the Casimir effect. Then, we discuss the phenomenological implications of the volume stabilization. The mass ratio between the field and the effective 4D Planck scale is almost characterized by the ratio equation (8), assuming the factor $\left(\mu m/M_6^2\right)^2$ takes an optimal value of $\mathcal{O}(1)$. The corresponding quantity in the 4D toy model is given by

$$R(\alpha, \delta_+, g_4) = \frac{g_4^2}{2\pi - \delta_+} \, \mathrm{e}^{-(A_4+B_4)/B_4}. \tag{14}$$

In figure 3 we have plotted $\log_{10}[R(\alpha, \delta_+ = 0.01, g_4)]$ as a function of α. We obtain larger mass hierarchies for smaller α and smaller g_4. In the limit $\alpha \to 1$, these curves converge at the same point. Numerical plots also show that this feature is almost independent of the value of the deficit angle δ_+ (or ω).

4. Volume stabilization in 6D warped compactifications

We discuss the stability of the volume modulus against the quantum corrections. As mentioned, we have no knowledge on the conical contribution of a_6. However, it is possible to perform a summation of modes given by equation (13) to evaluate $\zeta(0, \Delta_{6,\epsilon=0})$ instead of the direct evaluation of $a_6(f = 1)$. In the 4D model, we confirmed that the identity $a_4(f = 1) = \zeta_4(0)$ is satisfied by our mode-sum scheme and thus we believe that it is also true for the 6D model.

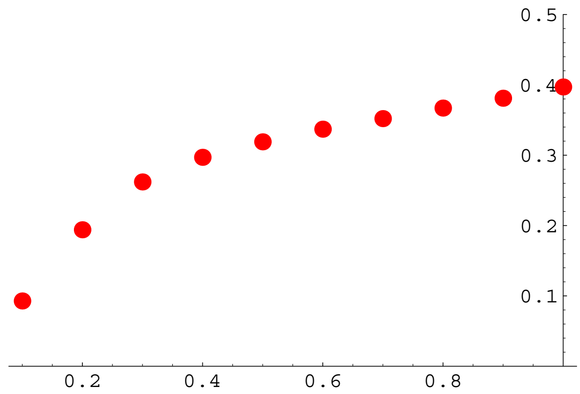

Figure 4. A plot for critical warping $\alpha_*(\omega)$ is shown as a function of ω in the 6D model.

The extention of the way of our mode summation developed in the analysis of 4D model to 6D is straightforward.

As a result, an important observation is that for smaller α the sign of the integrand of $\zeta(0, \Delta_{6,\epsilon=0})$ becomes negative implying that the volume modulus is *destabilized*. In the case of 4D toy model discussed in [8], there is also negative brane contribution but then the bulk effect still dominates and the modulus is always stabilized. For any value of ω, we obtain the critical value of $\alpha_*(\omega)$, below which the volume modulus is destabilized. In figure 4, we show the critical α_* as a function of ω. Note that the value of α_* does not depend on g_6, since $B_6 \propto g_6^4$. In discussing phenomenological implications, we need to know the cocycle function and a_6. So for now we leave this issue for future studies.

5. Summary

In this paper, we discussed volume stabilization in warped compactification model in 6D Nishino–Sezgin supergravity, whose internal space is bounded by codimension two branes (conical singularities). Due to the scale invariance of the underlying theory, the volume modulus appears in the 4D effective theory.

Because of the lack of the formulation of the conical heat kernel in the 6D spacetime, we first discussed the possibility of the volume stabilization in the 4D version of the warped compactification. We presented an exact mode summation and evaluation of conical contribution. As a result, we expect a larger mass hierarchy for smaller α (where α characterize the warping) and for large gauge coupling constant g_4. Indeed, for $\alpha \ll 1$, the contribution from the cocycle function becomes important and gives rise to a large mass hierarchy on the brane, though at the same time the effective mass of the modulus may become lighter. The curves for different values of g_4 in figure 3 converge to the same point at $\alpha = 1$. Similar results are obtained in the case of the vacuum (Casimir) energy density on the branes.

Then, we discussed the volume stabilization in the original 6D model. As a result, especially for $\alpha < \alpha_*(\omega)$, where $\alpha_*(\omega)$ is deficit angle dependent critical value (and does not depend on the bulk gauge coupling g_6), the volume modulus is destabilized because of strong negative contribution from the branes.

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